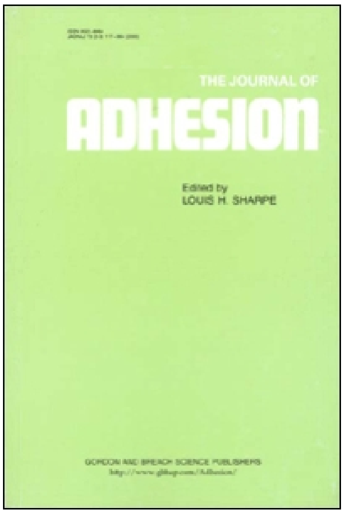

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The Peninsula Blister Test: A High and Constant Strain Energy Release Rate Fracture Specimen for Adhesives†

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An extension of the island blister developed by Allen and Senturia to a peninsula-like geometry produces a fracture test which retains the very high strain energy release rates which are possible with the island blister, and also results in a constant strain energy release rate test for adhesive bonds. Analytical solutions are provided for predicting the strain energy release rates for this peninsula blister specimen when the blister adherend may be considered a plate, a pre-stressed membrane, and a simple membrane. Preliminary experimental results for a PSA tape are also presented. The analytical results suggest that the specimen may be extended to a variety of practical adhesive systems.

KEY WORDS Adhesive fracture test; blister test; peninsula blister test; pressure sensitive adhesives; constant strain energy release rate fracture specimen.

INTRODUCTION

A number of fracture tests have been proposed for measuring the fracture toughness of a variety of adhesive bond systems. For systems where the adherends are sufficiently stiff, a number of tests may be used, including the double cantilever beam specimen developed by Mostovoy and Ripling,[1] the Outwater double torsion specimen,[2] the cracked lap shear test,[3] and the cone and precracked single lap shear specimens advocated by Anderson and his associates.[4] For situations where one or both of the adherends are soft, thin, or flexible, alternate test geometries must be used. Again, a number of techniques are available, including the blister geometry applied to membranes by Gent,[5] the constrained blister specimen proposed by Dillard *et al.*[6,7] and Moet *et al.*[8] and analyzed by Lai and Dillard,[9,10] the tape pull-off test proposed by Gent,[11] the flawed membrane test advocated by Farris,[12] and the island blister test proposed

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by Senturia and Allen.[13,14] This latter technique offers a significant advantage over other techniques for systems with strong adhesion and relatively weak membranes because extremely high strain energy release rates can be generated at relatively low pressures and membrane stresses. A variety of peel specimens have also been used for testing adhesive bonding of thin, flexible adherends, but these tests are often considered to be less satisfactory than fracture tests because of the the large deformations which often develop.

The blister test was originally proposed by Dannenberg[15] for measuring the adhesion of coatings. To have better control over the debond growth, he confined the blister to form in a narrow groove, thereby resulting in a constant strain energy release rate specimen. Williams[16] utilized a circular debond to measure the fracture energy of elastomeric materials adhesively bonded to a rigid substrate. Blister specimens are quite versatile and have been applied in a number of configurations as mentioned above. Blister tests have been applied to a wide variety of adhesive systems, including paints, coatings, elastomers, bonded plates,[17] pressure sensitive adhesive tapes, and even adhesion to ice.[18] The standard blister is quite compatible with environmental exposures because the pressurizing medium is contained within the blister region. For circular versions of the blister specimen, the axisymmetric shape minimizes problems associated with edge effects of finite width specimens, and diffusion perpendicular to the debond front eliminates spurious effects for environmental exposure. One of the disadvantages with the standard blister is that the strain energy release rate increases with the fourth power of the debond radius, thus making accurate evaluation of the debond essential and resulting in a very unstable fracture specimen. To minimize this problem, the constrained blister[6–10] was introduced. While the constrained blister specimen is a constant strain energy release rate specimen only under very limiting cases, it does significantly reduce the dependence of the strain energy release rate on debond radius. Nonetheless, the constrained blister specimen has encountered several problems. Questions have arisen regarding the effect of friction between the adherend and the constraint, and although numerical results suggest that the effect is totally negligible,[9] the difficulty of analyzing a contact problem with large deflections causes concern. The peninsula blister proposed herein seems to overcome many of the difficulties in analysis, and offers a truly constant strain energy release rate test over a relatively large test section.

THE PENINSULA BLISTER SPECIMEN

The island blister specimen proposed by Allen and Senturia[13,14] derives its high strain energy release rate from the fact that the debond front is reduced to a very small length. A moderate increase in compliance is produced by a relatively small increase in debond area, thereby giving rise to large strain energy release rates. As the membrane attachment site decreases in radius, the calculated strain energy release rates increase without bound. In an effort to extend the concept of this technique to other adhesive systems, we have introduced the peninsula blister

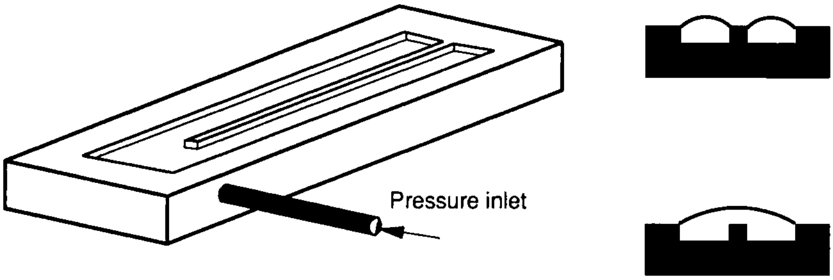

FIGURE 1 A diagram of the peninsula blister specimen substrate and cross-sections of the blister adherend for bonded and debonded sections.

specimen, a natural extension of the island blister to a larger "geographical" feature.[19] The name derives from the fact that debonding occurs along a narrow "peninsula" which extends into the blister region, as shown in Figure 1. Being similar to the island blister specimen, the peninsula blister retains the advantageous high strain energy release rate for any given pressure, a prerequisite for testing adhesion of thin, delicate films. Unlike the island blister, the peninsula blister strain energy release rate does not increase without bound as the debond progresses; this results in the highly desirable constant strain energy release rate nature of the peninsula blister specimen. Added features include the larger debond areas and additional data points that can be obtained from a single specimen. Disadvantages are that the fabrication of the specimen may be difficult for certain material systems, and that the specimen is no longer axisymmetric, thereby possibly reducing the utility for environmental exposure testing. There is a mixture of mode I and II for the energy release rates, but the nature of this mode mix has not yet been investigated.

Analysis reveals that the strain energy release rates obtainable for a given pressure are very high for the peninsula blister. In fact, the peninsula blister often exceeds the very high values obtainable with the island blister. In order to understand this beneficial aspect, it is instructive to recall that for fracture specimens which exhibit linear force *versus* deflection behavior, the strain energy release rate is obtained simply from:

$$G = \tfrac{1}{2}p^2 \frac{\partial C}{\partial A} \tag{1}$$

where p represents the generalized force (for pressurized blister geometries: pressure), C represents the generalized compliance (for blister geometries: displaced volume/pressure), and A represents the debond area. For a given pressure, the only two ways to increase G are to increase the increments in compliance produced by debonding, or to decrease the amount of debond area which is needed to produce a given amount of compliance. Each of these methods is frequently applied to various fracture geometries. For example, using more compliant adherends or longer initial debonds increases the compliance change for double cantilever beam specimens, whereas notching the beams to give

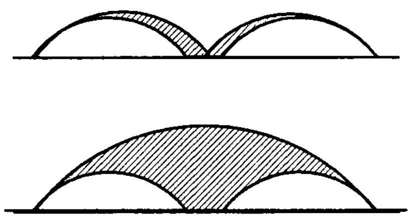

FIGURE 2 A schematic contrasting compliance changes possible with the island and peninsula blister specimens.

reduced bond widths reduces the denominator in Eq. (1). The high strain energy release rates for the island and peninsula blister specimens may also be explained in terms of Eq. (1). Because the island blister specimen is axisymmetric, the change in area needed to effect a change in compliance becomes vanishingly small as the island bond radius, b, approaches zero. The volume changes do not vanish as the bond radius decreases, so unbounded strain energy release rates are possible for membrane-like specimens.

The high strain energy release rates for the peninsula blister specimen arise from two factors. The peninsula width, $2b_0$, is kept small to reduce the debond area associated with a given increment in debond length. More importantly, however, the specimen takes advantage of the tremendous changes in compliance which result as debonding proceeds along the peninsula. A comparison of the compliance changes for the island and peninsula blister geometries may be seen in Figure 2. Although direct comparison is not possible because the island specimen is axisymmetric and the peninsula is a linear geometry, it is seen that the compliance of the latter can be much larger. The primary difference arises because the island blister ceases to be a fracture test once the island radius vanishes. A large compliance change occurs as the membrane displaces following complete debonding, but this large compliance change does not contribute to a fracture event. On the other hand, the peninsula blister is able to take advantage of this full compliance change as the debond proceeds along the peninsula in a self-similar manner. This produces very large strain energy release rates for any given pressure.

ANALYTICAL SOLUTIONS FOR THE PENINSULA BLISTER

Strain energy release rates for the peninsula blister on a rigid substrate may easily be determined analytically. This paper presents results for cases where the blister behaves as a plate, as a membrane with a dominant prestress, and as a membrane with no prestress. These will be compared with solutions for other blister geometries. For our purposes, we will assume that the peninsula blister length dimensions are much larger than the width dimensions in order to simplify the analysis. We have shown for the plate case that this restriction is easily satisfied by convenient specimen geometries.

Plate solution

If one assumes that the blister adherend has bending resistance and that the total deflections of the blister do not exceed the order of the thickness of the blister adherend, simple plate theory may be applied. The governing equation is

$$\nabla^4 w = -\frac{p}{D} \tag{2}$$

where ∇^4 is the biharmonic operator, w is the deflection of the plate, p is the applied pressure, and D is the plate rigidity given by

$$D = \frac{Et^3}{12(1-\nu^2)} \tag{3}$$

where E is Young's Modulus, t is the thickness, and ν is Poisson's ratio of the blister adherend. For a fully clamped plate of infinite length, the displacement across the width of the plate is easily shown to be

$$w(x) = \frac{px^2}{24D}(W^2 - 2Wx + x^2) \tag{4}$$

where W is the plate width, and p is a uniformly applied pressure. The compliance per unit length is

$$C = \frac{1}{p}\int_0^W w(x)\,dx = \frac{W^5}{720D} \tag{5}$$

Applying equation 1, we find that the strain energy release rate to debond an infinite strip uniformly is:

$$G = \frac{p^2W^4}{288D} \tag{6}$$

Thus, for the peninsula blister specimen shown in Figure 3, we can write the strain energy release rate for debonding at sites 2 and 3 as:

$$G_2 = \frac{p^2(a_0 - b_0)^4}{288D} \tag{7}$$

and

$$G_3 = \frac{p^2(2a_0)^4}{288D} = \frac{p^2a_0^4}{18D} \tag{8}$$

For small b_0, G_2 may be approximated by

$$G_2 \approx \frac{p^2a_0^4}{288D} \tag{9}$$

and we see that G_3 is always at least 16 times G_2. For debonding along the

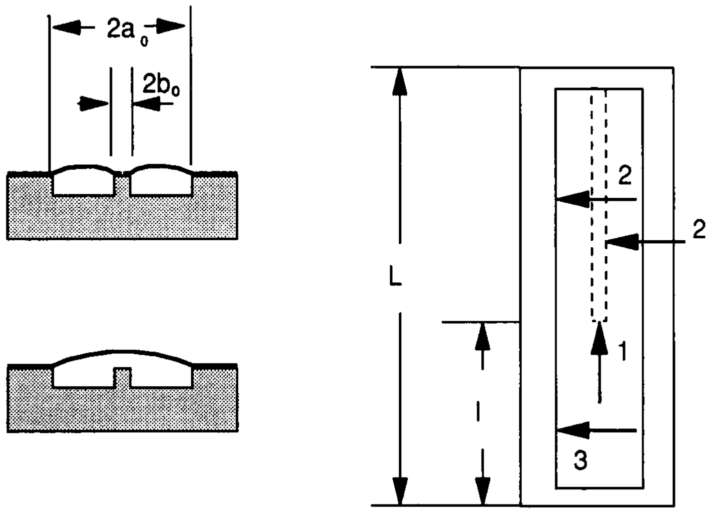

FIGURE 3 Possible debonding sites for the peninsula blister specimen.

peninsula, we note that the compliance per unit length at a bonded section is

$$C_{\text{bonded}} = 2\left[\frac{(a_0 - b_0)^5}{720D}\right] \tag{10}$$

and for the debonded section is

$$C_{\text{debonded}} = \frac{(2a_0)^5}{720D} \tag{11}$$

Letting l be the debonded length and L be the total length of the specimen, we can write the specimen compliance as

$$C_{\text{total}} = lC_{\text{debonded}} + (L - l)C_{\text{bonded}} \tag{12}$$

and apply equation 1 to obtain:

$$G_1 = \frac{p^2}{1440Db_0}[16a_0^5 - (a_0 - b_0)^5] \tag{13}$$

From this, we see that G_1 is always larger than G_2 implying that the debond along the peninsula should always occur along the length rather than across the width. It is noted that for $b_0/a_0 > 0.2$, debonding at site 3 is more likely than at site 1. This can easily be prevented by clamping the perimeter of the specimen to prevent debonding. To obtain the large strain energy release rates possible with this specimen, however, the peninsula width, $2b_0$, should be less than 20% of the total fixture width, $2a_0$. In Figure 4, the strain energy release rate has been normalized by p^2/D and plotted *versus* the normalized peninsula width for debonding at sites 1, 2, and 3. For comparison purposes, the island blister geometry with a plate adherend was analyzed using the numerical scheme developed by Lai and Dillard[10] and also plotted on the figure. A closed form plate solution for the island blister is also presented in Appendix A. In Figure 5,

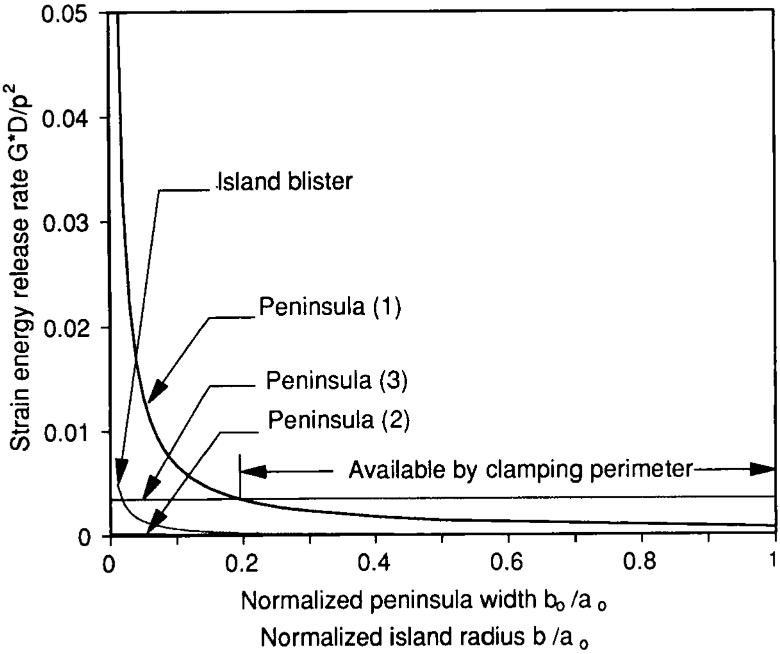

FIGURE 4 Effect of relative peninsula width on the strain energy release rate of the island and peninsula blister specimens: plate solution.

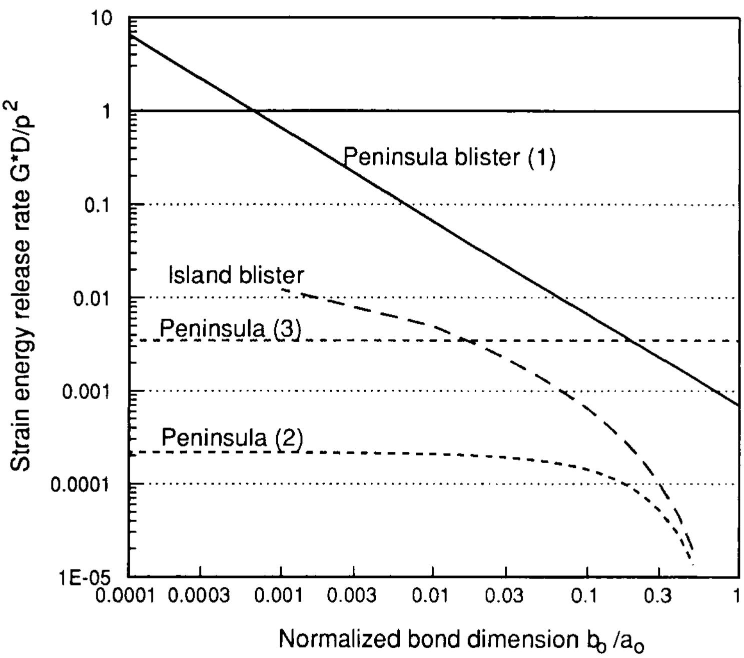

FIGURE 5 Effect of relative peninsula width on the strain energy release rate of the island and peninsula blister specimens: plate solution.

a log-log plot of the same information, reveals that for the plate solution, the peninsula blister always produces larger values of GD/p^2 than the island blister specimen of similar geometry. It should be noted that while the strain energy release rate for the peninsula blister is larger than for the island blister of similar cross sectional dimensions, the peninsula blister is considerably longer. If the length of the peninsula blister was required to be the same as the island blister diameter, the island blister could give a higher strain energy release rate.

Before leaving the plate solution, it is instructive to see under what conditions the infinite solution proposed above becomes approximately valid for finite length plates. Taking data from Timoshenko and Woinowsky-Krieger,[20] Figure 6 illustrates the effect of aspect ratio on the midpoint deflection for a fully clamped plate. For an aspect ratio of 2, the deviation from the infinite plate solution is only 2%. One can argue that if the aspect ratios of the end regions of the peninsula blister are greater than 2, deviations from the above derivations should be negligible. We, thus, neglect data obtained for $l < 4a_0$ at the end of initial debonding, and $L - l < 2(a_0 - b_0)$ at the end to which debonding proceeds. An optimum gap at the end of the peninsula tip is approximately $4a_0$, although a small gap may be desirable for some materials to allow a debond to begin propagation outside of the test section. Deviations from constant strain energy release rate within the remainder of the specimen should be completely negligible. Based on this reasoning, Figure 7 compares the manner in which the

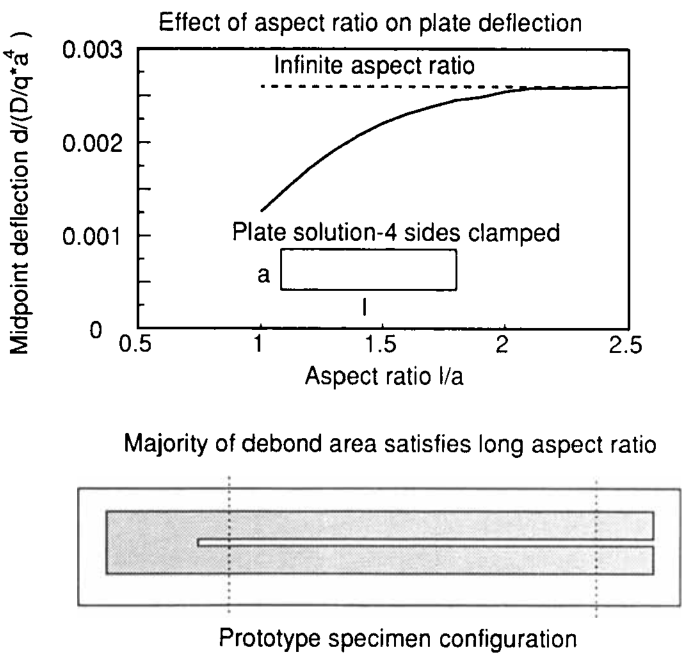

FIGURE 6 Influence of aspect ratio on the midpoint deflection of a rectangular plate and large central region of the prototype specimen with aspect ratios greater than 2.

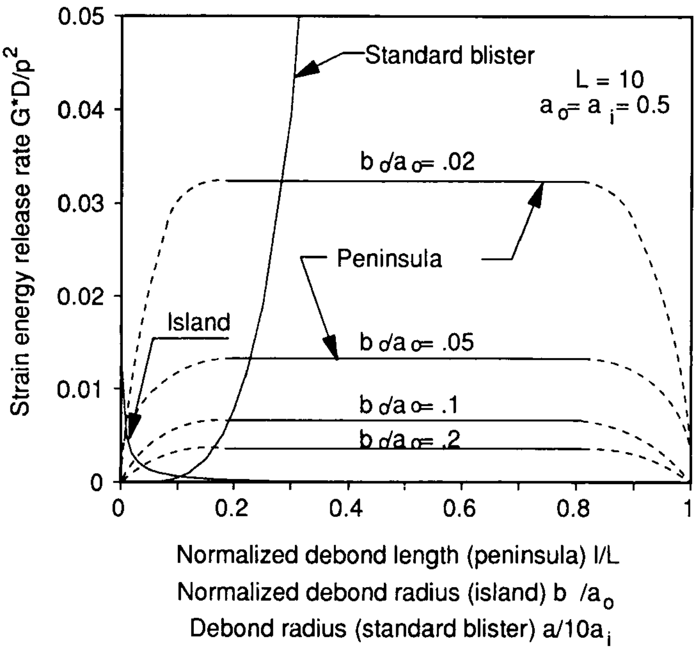

FIGURE 7 Variation in strain energy release rate with debond distance for the standard blister, island blister, and peninsula blister specimens: plate solution.

strain energy release rate varies for the standard, island, and peninsula blister as debonding proceeds. This suggests a significant advantage for the peninsula blister geometry.

Membrane with dominant prestress solution

Membrane solutions are more difficult than plate solutions because the load-deflection behavior is nonlinear, in general. An exception to this is the case where the membrane is initially prestressed in tension, and the applied pressures and resulting deflections are small enough that the additional membrane stresses induced are negligible when compared with the initial prestress. The governing equation is simply

$$\nabla^2 w = -\frac{p}{N} \tag{14}$$

where N is the prestress in force per unit length. Interestingly, no constitutive properties appear in this equation. Allen and Senturia[13] have solved this equation for the island blister, obtaining

$$G = \frac{p^2 b^2}{32N}\left[\frac{(\beta^2 - 1)}{\ln \beta} - 2\right]^2 \tag{15}$$

where $\beta = a_0/b$. For the peninsula blister, we again assume infinite length dimensions, and obtain the deflection:

$$w(x) = \frac{-px^2}{2N_x} + \frac{pWx}{2N_x} \tag{16}$$

where N_x is the membrane tension in the x direction. For long aspect ratios, the tension in the y direction, N_y, does not affect the solution away from the ends. The compliance per unit length is

$$C = \frac{W^3}{12N_x}$$

and the strain energy release rates for sites 2 and 3 are found to be

$$G_2 = \frac{p^2(a_0 - b_0)^2}{8N_x} \tag{17}$$

and

$$G_3 = \frac{p^2 a_0^2}{2N_x} \tag{18}$$

Following a similar procedure as used for the plate solution, we obtain the strain

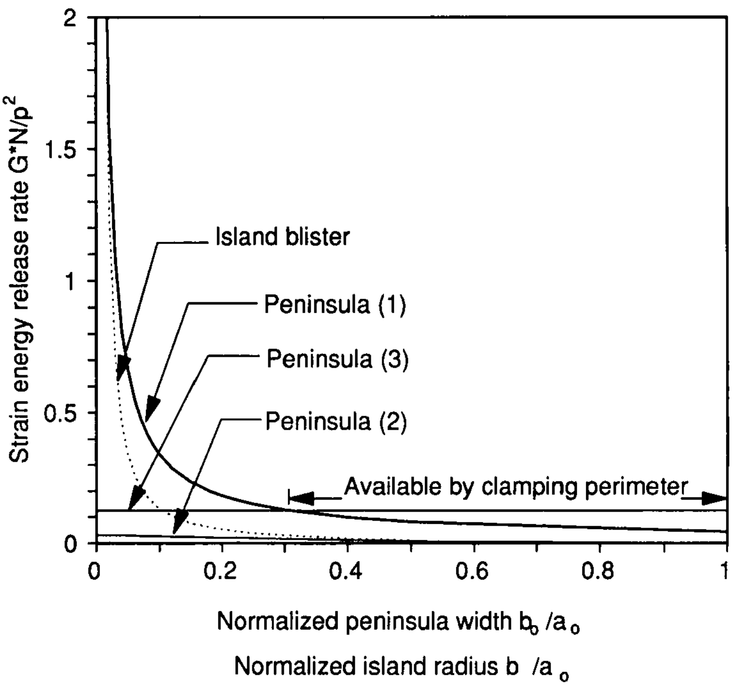

FIGURE 8 Effect of relative peninsula width on the strain energy release rate of the island and peninsula blister specimens: solution for membrane with prestress.

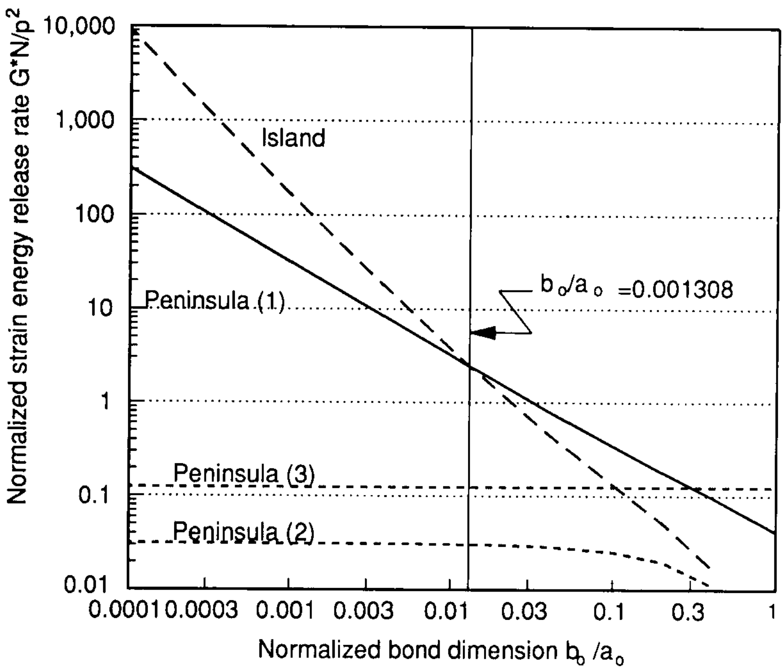

FIGURE 9 Effect of relative peninsula width on the strain energy release rate of the island and peninsula blister specimens: solution for membrane with prestress.

energy release rate for debonding along the length of the peninsula:

$$G_1 = \frac{p^2}{24N_x b_0}[4a_0^3 - (a_0 - b_0)^3] \tag{19}$$

Again, we find that debonding along the length of the peninsula is always favored over debonding across the width. We also find that debonding at site 1 is favored over site 3 for $b_0 < 0.3\,a_0$. These results and the results for the island blister are shown in Figures 8 and 9. It is interesting to note that for similar a and b values, the strain energy release rate for the peninsula blister exceeds that of the island blister until the debond radius of the island blister becomes less than $0.01308a_0$. Beyond this point, the island blister outperforms the peninsula blister because of the vanishingly small debond area. It is interesting to contrast the island blister behavior for the plate and prestress membrane solutions. The flattening of the plate solution in Figure 6 at very small debond radii is believed to come from the requirement that plate slopes must be continuous. Further reductions in small island radii do not permit additional specimen compliance. Membrane solutions do not require slope continuity and, hence, compliance changes are significantly larger. In actual applications, bending stiffness of thin "membranes" may not be negligible as bond radii become extremely small, so a deviation from Eq. (15) may be expected. A comparison of the strain energy release rate variation with debond growth is given for the prestress membrane case in Figure 10.

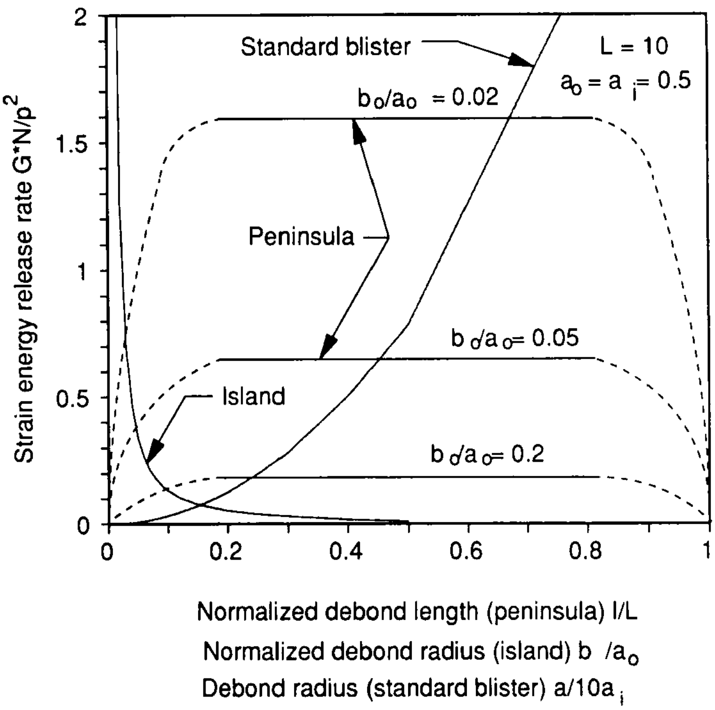

FIGURE 10 Variation in strain energy release rate with debond distance for the standard blister, island blister, and peninsula blister specimens: solution for membrane with prestress.

Membrane with no prestress solution

For the case of a membrane without a dominant prestress, the problem becomes nonlinear and exact solutions are very difficult to determine. For our purposes, we will here present a solution for the case where there is no initial prestress. The only stresses in the membrane are associated with the stretching of the film. For circular geometries, even this case does not yield simple closed form solutions, as is evidenced by the slightly different answers obtained by Gent[5] and Allen[13] for the same geometry. These differences arise because of differences in formulating the deflections for a circular blister. Fortunately, for the case of a non-prestressed rectangular blister of infinite aspect ratio, the solution for the deflection is unambiguous, and the results are provided below. Unlike the case of a dominant prestress, however, we find that the constitutive properties of the blister will now influence the deflections.

If we consider the case of a membrane of infinite length attached at supports a distance of W apart, and subject it to a pressure difference, we find that the membrane will deflect into a circular arc. The stress in the membrane is known from simple thin-wall pressure vessel theory to be

$$\sigma_x = \frac{pR}{t} \tag{20}$$

where R is the radius of curvature of the deflected membrane. Because the membrane is loaded in a plane-strain manner, the strain is given by

$$\varepsilon_x = \frac{\sigma_x}{E}(1 - \nu^2) \tag{21}$$

Using trigonometry, one can develop a relationship between the strain and the radius of curvature. For our purposes, the trigonometric functions have been expanded as Taylor series, and assuming small deflections, we have retained only the lower order terms. This will limit the applicability of the following derivations. Letting 2θ represent the subtended angle of the arc of the blister, it may be shown that:

$$\theta = \left[\frac{3pW(1 - \nu^2)}{Et}\right]^{1/3} \tag{22}$$

By relating the geometry and constitutive relations, we are able to write that the volume per unit length of this membrane is given simply by

$$V = \frac{W^{7/3}}{6}\left[\frac{3p(1 - \nu^2)}{Et}\right]^{1/3} + \frac{W^3}{6}\frac{p(1 - \nu^2)}{Et} \tag{23}$$

By applying the energy balance equation,

$$G\delta A = \delta W - \delta U$$

where δA is the variation of debond area, δW is the variation of external work done on the system, δU is the variation of stored elastic energy. Again we can determine that the strain energy release rates at sites 2 and 3 are given:

$$G_2 = \tfrac{7}{24}[p(a_0 - b_0)]^{4/3}\left[\frac{3(1 - \nu^2)}{Et}\right]^{1/3} + \tfrac{1}{4}[p(a_0 - b_0)]^2\left[\frac{(1 - \nu^2)}{Et}\right] \tag{24}$$

$$G_3 = \tfrac{7}{12}(pa_0)^{4/3}\left[\frac{6(1 - \nu^2)}{Et}\right]^{1/3} + (pa_0)^2\left[\frac{1 - \nu^2}{Et}\right] \tag{25}$$

Considering the volume per unit length of the bonded and debonded section, we can write the displaced volume of the specimen:

$$V_{\text{total}} = \frac{1}{6}\left[\frac{3p(1 - \nu^2)}{Et}\right]^{1/3}[l(2a_0)^{7/3} + 2(L - l)(a_0 - b_0)^{7/3}] + \frac{1}{6}\left[\frac{p(1 - \nu^2)}{Et}\right][l(2a_0)^3 + 2(L - l)(a_0 - b_0)^3] \tag{26}$$

Applying the original energy balance equation, the strain energy release rate at site 1 is:

$$G_1 = \frac{1}{16b_0}p^{4/3}\left[\frac{3(1 - \nu^2)}{Et}\right]^{1/3}[(2a_0)^{7/3} - 2(a_0 - b_0)^{7/3}] + \frac{1}{24b_0}\frac{p^2(1 - \nu^2)}{Et}[(2a_0)^3 - 2(a_0 - b_0)^3] \tag{27}$$

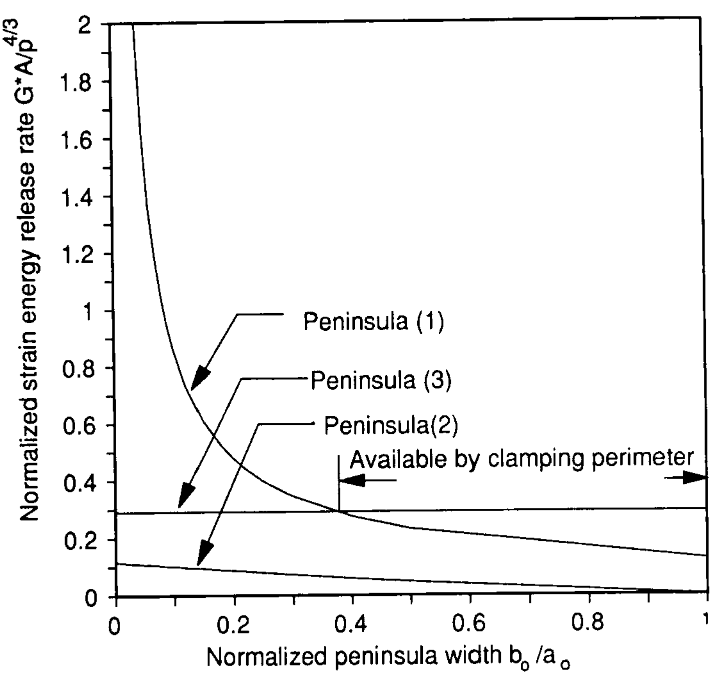

FIGURE 11 Effect of relative peninsula width on the strain energy release rate of the island and peninsula blister specimens: solution for membrane without prestress.

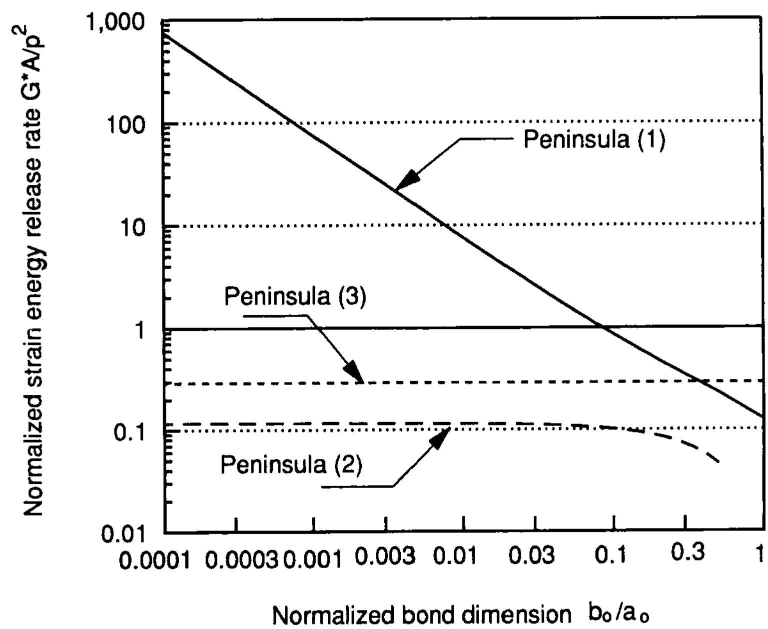

FIGURE 12 Effect of relative peninsula width on the strain energy release rate of the island and peninsula blister specimens: solution for membrane without prestress.

Because these equations are nonlinear and contain several terms, simple normalization is not possible unless we consider the special case where the stiffness of the membrane is sufficiently great that only the cube root term of the compliance is retained. Keeping only this term is most consistent with the small displacement assumptions made earlier, and is the same order of approximation as used by Gent.[5] Figures 11 and 12 illustrate the strain energy release rates for this case, where A is given by:

$$A = \left[\frac{Et}{3(1 - \nu^2)}\right]^{1/3}$$

PRELIMINARY EXPERIMENTAL RESULTS

Although we would eventually like to test adhesion for thin films and coatings, a preliminary fixture has been constructed for use with adhesive tapes. The fixture illustrated in Figure 1 consists of a base which is machined to provide the indicated pressurizing groove and leave the characteristic peninsula along which debonding occurs. The pressurizing medium is introduced into the groove by means of a fitting and internal hole through the substrate. An adhesive tape membrane is then pressed onto the substrate, bonding to the outer perimeter and to the peninsula. If the peninsula is narrow in comparison with the total width of the pressurizing groove, debonding will occur along the entire peninsula before proceeding across the perimeter region. For our current studies, a clamp is bolted over the top of the specimen to assure that debonding occurs along the perimeter, although this is probably not necessary for properly designed fixtures and consistently uniform adhesives. When pressure is introduced to the fixture,

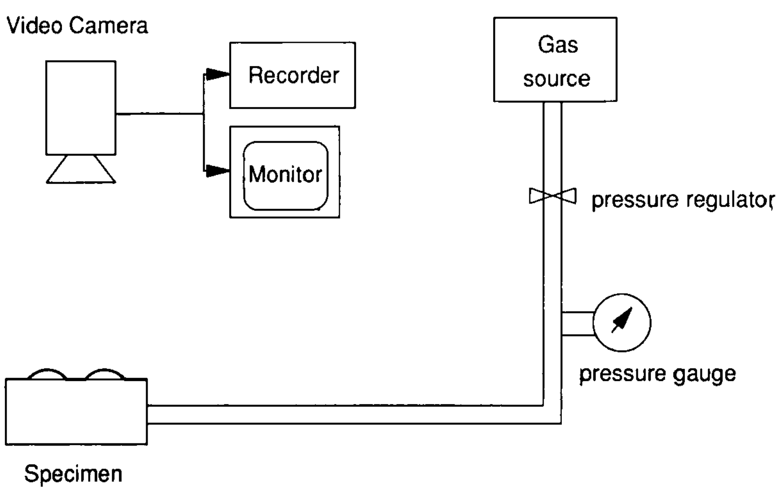

FIGURE 13 Schematic diagram of a peninsula blister test fixture.

TABLE I

E	t	ν	$2a_0$	$2b_0$
743 MPa	0.11 mm	0.36	12.85 mm	1.63 mm

the membranes flex as indicated in Figure 1. Debonding initiates at the tip of the membrane, and proceeds along the length of the peninsula. Debonding data are recorded with a video camera to determine debond rate, as is shown in Figure 13.

Tapes with polyester backing and rubber adhesive, whose properties are summarized in Table I were used in this study.

An important assumption in the derivation of the strain energy release rate for non-prestressed membrane case is to retain only the first two orders of the Taylor's series expansions. An effort has been made to estimate the relative errors of this omission and the error is less than 3% for the current example. This implies that the assumption is quite reasonable. Typical debonding results are presented in Figure 14. The debonding progresses at a fairly uniform rate, although a significant periodic nature is evident. This may be associated with regular variations in adhesive thickness, etc. Further tests are needed to evaluate this behavior and to extend this technique to other adhesive-adherend systems.

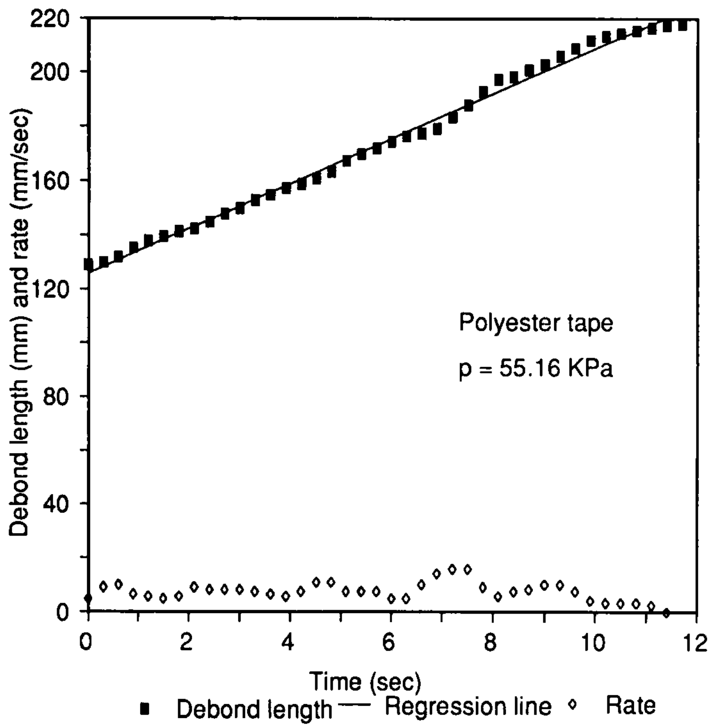

FIGURE 14 The debonding length and rate *vs.* time for a typical PSA polyester tape.

SUMMARY AND CONCLUSIONS

A new fracture specimen is proposed which offers very high strain energy release rates and maintains these rates at constant values over major portions of the debond length. This peninsula blister specimen is similar to the island blister in several aspects, but offers possible advantages for certain applications. Closed form solutions have been derived to predict the strain energy release rates for a plate adherend, a highly prestressed membrane adherend, and a membrane with no prestress. These solutions reveal that exceptionally high strain energy release rates are possible with this geometry. Simple calculations also suggest the high uniformity of the strain energy release rate as debonding proceeds. In summary, the peninsula blister test appears to offer several attractive features for measuring strain energy release rates. The high and constant strain energy release rate nature of the test make it worth investigating further. Specifically, we would like to investigate possible applications to membranes and coatings, and also use the specimen for measuring the debond energy for structural adhesives. The constant G nature also makes this specimen a candidate for fatigue tests on adhesives.

Further studies on the mode mix are needed to determine accurately appropriate values of critical strain energy release rate. The mode mix is likely to depend strongly on the appropriate governing equation and the geometric and constitutive properties of the system.

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APPENDIX

For the island blister, we can assume that the deflections of the blister are greatly less than the thickness of the adherend and the thickness of the adherend is also greatly less than the film radius. So, simple plate theory can be applied, and the governing equation can be expressed as:

$$\nabla^4 w = -\frac{p}{D} \tag{28}$$

The deflection is given as:

$$w(r) = C_1 + C_2 lnr + C_3 r^2 + C_4 r^2 lnr + \frac{pr^4}{64D} \tag{29}$$

In the case of an island blister, four boundary conditions are applied at the debond edge of the island b, and the outer radius a,

$$w(b) = 0, \qquad \frac{dw(b)}{dr} = 0$$

$$w(a) = 0, \qquad \frac{dw(a)}{dr} = 0 \tag{30}$$

The closed form solution was obtained using MACSYMA:[21]

$$C_1 = \frac{p}{64D}\frac{4a^2b^2(b^4\ln a - a^4\ln b)(\ln a - \ln b) + a^2b^2(a^2-b^2)^2[2(\ln a + \ln b) - 1]}{[2ab(\ln a - \ln b)]^2 - (a^2-b^2)^2}$$

$$C_2 = \frac{p}{16D}\frac{a^2b^2(b^4-a^4)(\ln b - \ln a) - a^2b^2(a^4+b^4) + 2a^4b^4}{[2ab(\ln a - \ln b)]^2 - (a^2-b^2)^2}$$

$$C_3 = \frac{p}{64D} \frac{\begin{array}{c}8a^2b^2[(b \ln a)^2 + (a \ln b)^2 - (a^2 + b^2) \ln a \ln b] \\ + (a^2 - b^2)^2[a^2(2 \ln a - 1) + b^2(2 \ln b - 1)]\end{array}}{[2ab(\ln a - \ln b)]^2 - (a^2 - b^2)^2}$$

$$C_4 = \frac{p}{32D} \frac{(a^2 - b^2)[a^4 - b^4 - 4a^2b^2(\ln a - \ln b)]}{[2ab(\ln a - \ln b)]^2 - (a^2 - b^2)^2} \quad \text{(31a,b,c,d)}$$

Substituting these expressions into Eq. (29) and then integrating with respect to debond area, we can obtain the compliance, that is:

$$\begin{aligned} C &= \frac{2\pi}{p} \int_b^a w(r) r \, dr \\ &= \frac{\pi}{2p} \left[\frac{p(a^6 - b^6)}{96D} + C_4 \frac{4(a^4 \ln a - b^4 \ln b) - (a^4 - b^4)}{4} \right. \\ &\left. + C_3(a^4 - b^4) + C_2(2a^2 \ln a - 2b^2 \ln b - a^2 + b^2) + 2C_1(a^2 - b^2) \right] \end{aligned} \quad (32)$$

Applying Equation (1), the strain energy release rate may be found from:

$$G = \tfrac{1}{2}(p^2) \frac{\partial C}{\partial A} = \frac{p^2}{4\pi} \frac{\partial C}{\partial b} \quad (33)$$